Problem 1

1) $$T_{el,max} = \frac{V^2}{2\omega_1\left[R_s \mp \sqrt{R_s^2 + (X_s + X_r')^2}\right]}$$

+ : motoring
− : braking

$$= \frac{V^2}{2\omega_1^2} \cdot \frac{1}{\left[R_s \mp \sqrt{R_s^2 + (X_s + X_r')^2}\right]} \cdot \omega_1 \qquad V/\omega_1 = \text{constant}$$

$$\gamma = \frac{T_{el,max}(f = 12Hz)}{T_{el,max}(f = 60Hz)} = \frac{R_s \mp \sqrt{R_s^2 + (2\pi \times 60)^2 (L_s + L_r')^2}}{R_s \mp \sqrt{R_s^2 + (2\pi \times 12)^2 (L_s + L_r')^2}} * \frac{12}{60}$$

$R_s = 0.074\ \Omega$
$L_s = L_r' = \frac{0.12}{2\pi \times 60}$

For motoring: $\gamma = \boxed{0.68}$

For braking: $\gamma = \boxed{1.46}$

2) At starting $\sigma = 1$

$$T_{el} = \frac{V^2}{\omega_1\left[(R_r' + R_s)^2 + (X_r' + X_s)^2\right]} = \frac{V^2}{\omega_1^2} \cdot \frac{1}{(R_s + R_r')^2 + (X_r' + X_s)^2} \cdot \omega_1$$

$$\frac{T_{el}(f = 12)}{T_{el}(f = 60)} = \frac{(R_s + R_r')^2 + (X_r' + X_s)^2\big|_{f=60}}{(R_s + R_r')^2 + (X_r' + X_s)^2\big|_{f=12}} \times \frac{12}{60} = \boxed{2.6}$$

$$I_r' = \frac{V/\sqrt{3}}{\sqrt{R_r'^2 + X_r'^2}}$$

$$\frac{I_r'(f = 12)}{I_r'(f = 60)} = \frac{V(f = 12)}{V(f = 60)} \times \sqrt{\frac{R_r'^2 + (2\pi \times 60)^2 L_r'^2}{R_r'^2 + (2\pi \times 12)^2 L_r'^2}} = \boxed{0.72}$$

problem:

1) $T_{el,r} = \frac{3 I_{r,r}'^2 R_r'}{\sigma \omega_r}$

$\sigma_r = \frac{n_s - n_m}{n_s}$

$n_s = \frac{120}{P} f_e = \frac{120}{6}(50) = 1000 \text{ rpm}$

$\sigma_r = \frac{1000-960}{1000} = 0.04$

$\omega_r = 1000\left(\frac{\pi}{30}\right) = 104.7 \text{ rad/sec}$

$I_{r,r}' = \left|\frac{Z_m}{Z_m + Z_r'}\right| I_{s,r}$

$I_{s,r} = \frac{400/\sqrt{3}}{|Z_{in}|}$

$Z_{in} = Z_s + \frac{Z_r' Z_m}{Z_r' + Z_m}$

$Z_s = R_s + jX_s = 1.55\angle 75^\circ \ \Omega$

$Z_r' = \frac{R_r'}{\sigma} + jX_r' = 5.27\angle 16.7^\circ \ \Omega$ $\Rightarrow Z_{in} = 6\angle 27^\circ \ \Omega$

$Z_m = j30 \ \Omega$

$I_{s,r} = \frac{400/\sqrt{3}}{6} = 38.5 A$

$I_{r,r}' = 36.22 A \Rightarrow E_r = |Z_r'| I_{r,r}' = 189 V$

$\Rightarrow T_{el,r} = \frac{3(36.22)^2(0.2)}{0.04(104.7)} = 188 \text{ N.m}$

Now, $T_{el} = 2P\lambda_m^2 \dfrac{R_r'/(p\omega_r)}{(R_r'/p\omega_r)^2 + L_r'^2}$

$T_{el} = \dfrac{188}{2}$

$p = 6$

$R_r' = 0.2\ \Omega$

$L_r' = \dfrac{1.5}{2\pi(50)}\ H$

$\lambda_m = L_m I_m$, $I_m = \dfrac{E_r}{\omega_e L_m} \Rightarrow \lambda_m = \dfrac{E_r}{\omega_e} = \dfrac{189}{2\pi(50)} = 0.62\ W.A$

$\Rightarrow$ solve for $\sigma \Rightarrow \sigma = 0.0274$

~~At~~ $I_1 = I_r' + I_m$

$I_r' = \dfrac{E}{Z_r'}$, $I_m = \dfrac{E}{jX_m}$

taking E as a reference

At 25 Hz $\Rightarrow E = 0.5 \times 189 = 94.5\ V$

$Z_r' = \dfrac{R_r'}{\sigma} + jX_r'(25Hz) = 5.4\angle 8^\circ\ \Omega$

$\Rightarrow I_r' = \dfrac{94.5}{5.4\angle 8^\circ} = 17.5\angle -8^\circ\ A$

$I_m = \dfrac{E}{jX_m(25Hz)} = \dfrac{94.5}{j(0.5\times 30)} = 6.3\angle -90^\circ\ A$

$I_1 = 17.5\angle -8^\circ + 6.3\angle -90^\circ$

$\Rightarrow \boxed{I_1 = 19.85\ A}$.

2) slip speed in rpm at $T_{st,r}$ and $f_e = 50\,Hz$

$n_\sigma = \sigma n_f = 0.04(1000) = 40$ rpm

Since the torque-speed curve is straight line, n_σ at half $T_{st,r}$ is $n_{\sigma_2} = 0.5 * 40 = 20$ rpm

At 25 Hz $\Rightarrow n_f = \frac{25}{50}(500) = 500$ rpm

$\sigma_2 = \frac{500-480}{500} = 0.04$

$I_s = I_m + I_r'$

$I_r' = \frac{E}{Z_r'}$, $I_m = \frac{E}{jX_m}$

taking E as a reference

$Z_r' = \frac{R_r'}{\sigma} + jX_r'(25\,Hz) = 5.06\angle 8.5^\circ$ A

$I_r' = \frac{94.5}{5.06\angle 8.5^\circ} = 18.7\angle -8.5^\circ$ A

$I_m = \frac{94.5}{j(0.5(20))} = 6.3\angle -90^\circ$ A

$I_s = 18.7\angle -8.5^\circ + 6.3\angle -90^\circ$

$\boxed{I_s = 20.6\ A}$

3) $n_\sigma = -40$ rpm

$n_f = n_m + n_\sigma = 800 - 40 = 760$ rpm

$n_f = \frac{120}{6} f_e \Rightarrow$ $\boxed{f_e = 38\,Hz}$.

At 38 Hz $\Rightarrow E = \frac{38}{50} \times 189 = 143.64\ V$

$\sigma = \frac{-40}{760} = -0.0526$

$Z_r' = \frac{R_r'}{\sigma} + jX_r'(38Hz) = 3.97\angle 163.3^\circ\ \Omega$

Taking E as a reference

$I_r' = \frac{E}{Z_r'} = 36.2\angle -163.3^\circ\ A$

$I_s = 36.2\angle -163.3^\circ + 6.3\angle -90^\circ$

$\boxed{I_s = 38.52\angle -154^\circ\ A}$.

$V = Z_s I_s + E = \left(0.4 + j1.5 \times \frac{38}{50}\right)(38.52\angle -154^\circ) + 143.64$

$\boxed{V = 156\angle -17.3^\circ\ V}$

Problem 3

$n_s = \frac{120}{P} f_e = \frac{120}{6} \times 60 = 1200$ rpm

$\omega_s = 1200\left(\frac{\pi}{30}\right) = 125.66$ rad/sec

$\sigma = \frac{1200-1164}{1200} = 0.03$

Without rotor resistance control,

$$T_{st} = \frac{V^2 R_r'/\sigma}{\omega_s\left[\left(R_s+\frac{R_r'}{\sigma}\right)^2+(X_s+X_r')^2\right]} = 78.5 \text{ N.m}$$

1.) at the breakdown torque

$$\frac{R'_{r,new}}{\sigma} = \sqrt{R_s^2+(X_s+X_r')^2}$$

when the breakdown occurs at standstill

$$R'_{r,new} = \sqrt{R_s^2+(R_s+X_r')^2}$$

$$= \sqrt{(0.4)^2+(1.6+1.6)^2} = 3.6\ \Omega$$

$$R'_{r,new} = R_r' + R_{eq}^{*'} + R'_{y,eq}$$

$$R'_{y,eq} + R_{eq}^{*'} = 3.6 - 0.6 = 3\ \Omega$$

$$R_{y,eq} + R_{eq}^{*} = \frac{3}{a_r^2} = 0.48\ \Omega$$

$$R_{y,eq} = 0.5 R_y = 0.5(0.02) = 0.01\ \Omega$$

$\Rightarrow R_{eq}^* = 0.47\ \Omega \Rightarrow R_{eq}^* = 0.5\left(1-\overset{0}{\cancel{\delta}}\right)R \Rightarrow \boxed{R = 0.94\ \Omega}$

2.)

$$T_{ex} = \frac{V^2 R'_{r,new}/\sigma}{\omega_r\left[\left(\frac{R'_{r,new}}{\sigma} + R_s\right)^2 + (X'_r + X_s)^2\right]} = 1.5 \times 78.5$$

$$\sigma = \frac{1200-960}{1200} = 0.2$$

solve for $R'_{r,new}$

$$\Rightarrow R'_{r,new} \approx 2.3\ \Omega$$

$$R'_r + R'_{x,eq} + R^{*\prime}_{eq} = 2.3\ \Omega$$

$$R'_{x,eq} + R^{*\prime}_{eq} = 1.7\ \Omega$$

$$R_{x,eq} + R^{*}_{eq} = 1.7/a_r^2 \doteq 0.272\ \Omega$$

$$0.01 + 0.5(1-\delta)R = 0.272$$

$$0.01 + 0.5(1-\delta)(0.94) = 0.272 \Rightarrow \delta = 0.4$$

3.) $R'_{r,new} = R'_r + R'_{x,eq} + R^{*\prime}_{eq}$

$$= 0.6 + a_r^2\left(R_{x,eq} + 0.5(1-\delta)R\right)$$

($R_{x,eq} = 0.5 R_x$)

$$= 0.6 + (2.5)^2\left[0.5(0.02) + 0.5(0.4)(0.94)\right]$$

$$= 1.84\ \Omega$$

$$1.5 \times 78.5 = \frac{(400)^2\, 1.84/\sigma}{125.66\left[\left(\frac{1.84}{\sigma} + 0.4\right)^2 + (3.6)^2\right]}$$

solve for $\sigma \Rightarrow \sigma \approx 0.15$

$$n_m = (1-\sigma)n_s = (1-0.15)(1200) = 1020\ \text{rpm}$$

Problems

$\omega_1 = \frac{\omega_e}{P} = \frac{2\pi \times 60}{3} = 125.7$ rad/sec

Rated torque

$T_{e,r} = \frac{1100\times10^3}{125.7} = 8751$ N.m

$I_m' = \frac{6600/\sqrt{3}}{jX_s} = 105.8\angle{-90^\circ}$ A

1.) $\sqrt{3}(6600) I_A = 1100\times10^3 + 3(1.2) I_A^2$

$\Rightarrow I_A = 99\angle{0^\circ}$ A

$I_f' = I_m' - I_A = 144.9\angle{-133^\circ}$ A

$I_F = \frac{X_s}{X_m}\frac{I_f'}{n} = \frac{76}{30}\frac{144.9}{2} = 86.94$ A

$\sin\delta = \frac{P_m}{3V I_f'} = \frac{1100\times10^3}{3\left(\frac{6600}{\sqrt{3}}\right)(144.9)} = 0.66$

$\delta = 41.6^\circ$

2.) $T_{e,max} = \frac{3V I_f'}{\omega_1} = \frac{3\left(\frac{6600}{\sqrt{3}}\right)(144.9)}{125.7} = 13178$ N.m

3.) $\frac{13178}{2} = \frac{3V I_f'}{\omega_1}\sin\delta = \frac{3\left(\frac{6600}{\sqrt{3}}\right)}{125.7}\times 144.9 \sin\delta$

$\Rightarrow \sin\delta = 0.33$

$I_A \sin\theta = -I_m' + I_f'\cos\delta = 30.4$

$I_A \cos\theta = I_f'\sin\delta = 47.8$

$I_A = \sqrt{30.4^2+47.8^2} = 56.6$ A

$pf = \cos\theta = \frac{47.8}{56.6} = 0.84$ (leading)

$$P_m = 4375.5 \times 125.7 = 550\ kW$$

$$P_{Losses} = 3(56.6)^2(1.2) + (86.94)^2(5) = 49.32\ kW$$

$$\eta = \frac{550}{550 + 49.32} = 0.92$$

4.) $\sqrt{3}(6600) I_A = 550 \times 10^3 + 3 I_A^2 \times 1.2$

$\Rightarrow I_A = 48.5\,A$

$I_A = I_f' \sin\delta = 48.5$

$I_m' = I_f' \cos\delta = 105.8$

$I_f' = \sqrt{(48.5)^2 + (105.8)^2} = 116\,A.$

$I_F = \frac{X_s}{X_m} \times \frac{I_f'}{n} = \frac{36}{30}\left(\frac{116}{2}\right) = 69.6\,A.$

problem 6

From problem 5

$\Rightarrow I_m' = 105.8\,A$

$PF = 1.0$

$T_{ey,r} = 8751\ N.m$

$\delta = 41.6°$

$I_{A,r} = 99\,A$

$I_F' = 144.9\,A$

$\omega_r = 1200\left(\frac{\pi}{30}\right) = 125.7$ rad/sec

1.) $I_A = 99\,A$

$\delta = 41.6°$

$PF = 1.0$

2.) $4375.5 = \frac{3(0.5 \times 36)}{0.5 \times 125.7} \times 105.8 \times 144.9 \times \sin\delta$

$\Rightarrow \sin\delta = 0.33$ or $\cos\delta = 0.94$

$I_A \cos\theta = -105.8 + 144.9(0.94) = 30.4$

$I_A \sin\theta = 144.9(0.33) = 47.8$

$I_A = \sqrt{(30.4)^2 + (47.8)^2} = 56.6\,A$

$\cos\theta = 0.54$ (leading)

3.) $P_m = 1100 \times 10^3\ W$

$\omega_r = 1500\left(\frac{\pi}{30}\right) = 157.1$ rad/s

$T_{el} = \frac{1100 \times 10^3}{157.1} = 7003\ N.m$

$X_s = \frac{1500}{1200} \times 36 = 45\ \Omega$

$$I_m' = \frac{V}{X_s} = \frac{6600/\sqrt{3}}{45} = 84.68\,A$$

$$I_f' \cos\delta = 84.68$$

$$I_f' \sin\delta = 99$$

$$I_f' = 130\,A$$